THE PROPAGATION OF RADIATION IN CAPILLARY-POROUS COLLOIDAL BODIES

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We present a solution for the problem of the distribution of spectral radiation fluxes through the thickness of a layer and the quantity of radiant energy absorbed by the elementary layers at a specific depth in capillary-porous colloidal bodies. We have demonstrated that the solution is found to be in good agreement with experiment.

The calculation of the external and internal heat and mass transfer in the drying and heat treatment of capillary-porous colloidal bodies by means of infrared (IR) radiation is impossible without information on the spectral radiation flows within the layer, nor without data on the spectral thermal-radiation characteristics (transmissivity T_λ, reflectivity R_λ, absorbtivity A_λ, as well as the emissivity ε_λ). Real bodies exhibit selective optical properties characterized by at least two spectral coefficients—absorption a_λ and scattering s_λ.

Until now, to account for the quantity of IR radiant energy absorbed at some depth of the material, we used either the Bouguer law, or the Bouguer–Lambert law [2]. However, a fundamental error arises in the utilization of these laws for the indicated purpose.

Let us examine the process involved in the monochromatic transport of a radiant flow in a plane layer (of thickness l) of a uniformly selective attenuating medium characterized by the spectral coefficients a_λ and s_λ.

Diffused monochromatic flows of radiant energy—respectively equal to Q_1 and Q_2 (Fig. 1)—impinge on the two sides of the layer. Let us determine the distribution of the radiation flows within this layer, and we can then use this quantity to determine the remaining characteristics of the radiation field.

We know of two approaches to the solution of this problem [3]; these are based exclusively on the two constants a_λ and s_λ. These methods were developed for stacks of weak-absorption but nonscattering layers [4], for thin weak-absorption but scattering powder-like layers [5], and for semitransparent gaseous media [6]. According to one of these methods the layer is assumed to be continuous and a_λ and s_λ are the unit-thickness constants of absorption and scattering for the layer. According to the other method, the layer is assumed to consist of elementary layers whose thickness corresponds to the average dimensions of the crystal particles and a_λ and s_λ are the constants of absorption and reflection for these crystals.

For the solution of the stated problem it is advisable to employ the continuous-layer method initially proposed by Schuster [6] in studying the transfer of radiation in a plane layer of the atmosphere. This method was subsequently developed by Gurevich [5], Kubelka and Munk [7], Gershun [8], et al. The fundamental idea behind the Shuster method involves the splitting of the radiation field within the plane layer into two discrete flows moving in opposite directions.

Let us isolate the elementary layer dx at a depth x, with the discrete co-current flows q_+ and q_- impinging on this layer. In the general case of bilateral irradiation of the layer, if we restrict our consideration only to a single scattering, the co-current flows will be composed of the following:

$$q_+ = Q_1 T_x + Q_2 T_{l-x} R_x,$$
$$q_- = Q_2 T_{l-x} + Q_1 T_x R_{l-x}, \qquad (1)$$

where R_x and T_x are, respectively, the reflectivity and transmissivity of a layer of thickness x and R_{l-x} and T_{l-x} are, respectively, the reflectivity and transmissivity of a layer of thickness $(l-x)$.

The scalar magnitude of the summary vector for the radiant flux density at the depth x is given by

$$q_\lambda = q_+ + q_-$$
$$= Q_1 T_x + Q_1 T_x R_{l-x} + Q_2 T_{l-x} + Q_2 T_{l-x} R_x. \qquad (2)$$

The quantity of energy absorbed in the elementary volume of the medium (of thickness dx) per unit time at a depth x is uniquely defined by the divergence of q_λ in the x-direction.

It follows directly from (2) that the basic error in the application of the Bouguer and Bouguer-Lambert laws to the determination of the summary flux density lies in the fact that no consideration is given to the co-current radiant flux at the depth x, said flux reflected from the remaining thickness $(l-x)$ of the layer

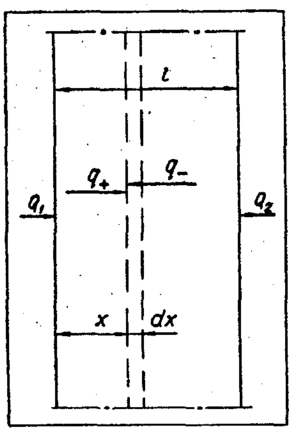

Fig. 1. Distribution of radiation fluxes in plane layer of selective absorbing and scattering medium.

or from x, and which is equal to $Q_1T_xR_{l-x} + Q_2T_{l-x}R_x$. Since most capillary-porous colloidal bodies exhibit substantial scattering coefficients [9] and, consequently, since they thus exhibit a high reflectivity (up to 90% in the near-IR region of the spectrum), the error is quite substantial (as much as 40%), particularly for the layers near the surface.

In real absorption and scattering media, a portion of each radiant flux incident on an elementary layer dx is absorbed ($a_\lambda dx$) and a portion is diffusely scattered ($s_\lambda dx$) in the direction opposite to that of the impinging flow (it is reflected). Then the differential equations characterizing the loss of the discrete interdependent radiant fluxes q_+ and q_- are written [7, 8] in the following form:

$$\frac{dq_+}{dx} = -(a_\lambda + s_\lambda)\, q_+ + s_\lambda q_-, \tag{3}$$

$$-\frac{dq_-}{dx} = s_\lambda q_+ - (a_\lambda + s_\lambda)\, q_-. \tag{4}$$

The boundary conditions are

$$q_+|_{x=0} = Q_1 \text{ and } q_-|_{x=l} = Q_2. \tag{5}$$

The solution for the system of equations (3)–(4) for the boundary conditions (5) has the form

$$q_+ = \frac{Q_1}{1-\Psi_\lambda^2}\left[\exp(-\sigma_\lambda x) - \Psi_\lambda^2 \exp(\sigma_\lambda x)\right] + \frac{Q_2\Psi_\lambda}{1-\Psi_\lambda^2}\left[\exp(\sigma_\lambda x) - \exp(-\sigma_\lambda x)\right], \tag{6}$$

$$q_- = \frac{Q_2}{1-\Psi_\lambda^2}\left[\exp\{-\sigma_\lambda(l-x)\} - \Psi_\lambda^2 \exp\{\sigma_\lambda(l-x)\}\right] + \frac{Q_1\Psi_\lambda}{1-\Psi_\lambda^2}\left[\exp\{\sigma_\lambda(l-x)\} - \exp\{-\sigma_\lambda(l-x)\}\right], \tag{7}$$

where

$$\Psi_\lambda = \eta_\lambda \exp(-\sigma_\lambda l), \tag{8}$$

$$\eta_\lambda = \frac{a_\lambda + s_\lambda - \sigma_\lambda}{s_\lambda} \text{ and } \sigma_\lambda = \sqrt{a_\lambda(a_\lambda + 2s_\lambda)}. \tag{9}$$

Equations (6) and (7) determine the monochromatic radiant fluxes impinging at a depth x on a layer of thickness l. These equations are simpler in form than those derived by Andrianov [10] for integral flows in a nonselective gaseous medium.

The quantity q_λ is determined from the equation

$$q_\lambda = \frac{Q_1}{1-\Psi_\lambda^2}(1+\eta_\lambda)\left[\exp(-\sigma_\lambda x) - \frac{\Psi_\lambda^2}{\eta_\lambda}\exp(\sigma_\lambda x)\right] + \frac{Q_2\Psi_\lambda}{1-\Psi_\lambda^2}\,\frac{(1+\eta_\lambda)}{\eta_\lambda} \times \left[\exp(\sigma_\lambda x) - \eta_\lambda \exp(-\sigma_\lambda x)\right]. \tag{10}$$

The quantity of energy absorbed at the depth x by an elementary volume of thickness dx per unit time, with consideration of (3) and (4), is given by

$$\frac{dq_\lambda}{dx} = \frac{dq_+}{dx} + \frac{dq_-}{dx} = -a_\lambda(q_+ + q_-). \tag{11}$$

Using (6), (7), and (11), we obtain

$$\frac{dq_\lambda}{dx} = \frac{a_\lambda Q_1}{1-\Psi_\lambda^2}(1+\eta_\lambda)\left[\exp(-\sigma_\lambda x) - \frac{\Psi_\lambda^2}{\eta_\lambda}\exp(\sigma_\lambda x)\right] + \frac{a_\lambda Q_2\Psi}{1-\Psi_\lambda^2}\,\frac{(1+\eta_\lambda)}{\eta_\lambda} \times \left[\exp(\sigma_\lambda x) - \eta_\lambda \exp(-\sigma_\lambda x)\right]. \tag{12}$$

The expressions with Q_1 and Q_2 are functions exclusively of the thickness of the layer for the given material and wavelength. Consequently, we can write

$$\frac{dq_\lambda}{dx} = a_\lambda Q_1 C_1(x) + a_\lambda Q_2 C_2(x). \tag{13}$$

The functions $C_1(x)$ and $C_2(x)$ are spectral distribution functions for the radiant energy absorbed through the thickness, and these can be determined in advance if we know the optical properties of the material.

To test the derived expressions characterizing the distribution of the monochromatic radiant fluxes within the capillary-porous colloidal bodies, as well as to test the hypothesis of linearity for the spectral absorbtion coefficients a_λ and the backscattering coefficients s_λ, we have to establish the relationship between the optical properties of these bodies and their thermal-radiation characteristics R_λ and T_λ, i.e., characteristics which can be measured.

We can determine this relationship from (6) and (7). From (7), under the condition of unilateral irradiation, we have

$$R_\lambda = \frac{q_-\big|_{\substack{Q_2=0\\ x=0}}}{Q_1} = \frac{\eta_\lambda[1-\exp(-2\sigma_\lambda l)]}{1-\eta_\lambda^2\exp(-2\sigma_\lambda l)}. \tag{14}$$

For the limit cases $a_\lambda = 0$ and $s_\lambda = 0$ formula (14) is in agreement with the familiar expressions for these cases:

$$R_\lambda|_{a_\lambda=0} = \frac{s_\lambda l}{1+s_\lambda l} \text{ and } R_\lambda|_{s_\lambda=0} = 0. \tag{15}$$

From (6), given the condition of unilateral irradiation, we have

$$T_\lambda = \frac{q_+\big|_{\substack{Q_2=0\\ x=l}}}{Q_1} = \exp(-\sigma_\lambda l)\,\frac{1-\eta_\lambda^2}{1-\eta_\lambda^2\exp(-2\sigma_\lambda l)}. \tag{16}$$

For the limit cases (16) is in agreement with the familiar relationships

$$T_\lambda|_{a_\lambda=0} = \frac{1}{1+s_\lambda l} \text{ and } T_\lambda|_{s_\lambda=0} = \exp(-a_\lambda l). \tag{17}$$

Formulas of the form of (14) and (16) were initially derived by Stokes [4] for a stack of weak-absorption nonscattering plates, and later on by Gurevich [5] for a layer of scattering weak-absorption ($a_\lambda l \ll 1$) particles in a vacuum.

With consideration of (8), (9), (14), and (16), it follows from (12) and (13) that the functions $C_1(x)$ and $C_2(x)$ at the boundaries of the layer assume the following values:

$$C_1(x)|_{x\to 0} = (1+R_\lambda) = C_2(x)|_{x\to l},$$
$$C_2(x)|_{x\to 0} = T_\lambda = C_1(x)|_{x\to l}.$$

Consequently, the surface layer is heated by two flows—the incident flow and the one which is reflected.

We can supply the Bouguer-Lambert law only to the layer with the coordinate $x = l$ when we have unilateral irradiation and $s_\lambda \ll 0.1$.

The fraction of energy absorbed by the layer can be found from the familiar relationship

$$A_\lambda = 1-(R_\lambda+T_\lambda) = 1-\frac{1+\eta_\lambda \exp(-\sigma_\lambda l)}{1+\eta_\lambda \exp(-\sigma_\lambda l)}. \quad (18)$$

For the limit cases we have the familiar relationships

$$A_\lambda|_{a_\lambda=0} = 0 \text{ and } A_\lambda|_{s_\lambda=0} = 1-\exp(-a_\lambda l). \quad (19)$$

On the basis of formulas (14), (16), and (18) we determine the spectral thermal-radiation characteristics of an optically infinitely dense layer ($\sigma_\lambda l \to \infty$) [7]:

$$R_{\lambda\infty} = \frac{a_\lambda + s_\lambda - \sigma_\lambda}{s_\lambda} = 1+\frac{a_\lambda}{s_\lambda} - \sqrt{\frac{a_\lambda^2}{s_\lambda^2} + 2\frac{a_\lambda}{s_\lambda}} = \eta_\lambda, \quad (20)$$

$$T_{\infty\lambda} = 0 \text{ and } A_{\infty\lambda} = \sqrt{\frac{a_\lambda^2}{s_\lambda^2} + 2\frac{a_\lambda}{s_\lambda}} - \frac{a_\lambda}{s_\lambda}. \quad (21)$$

Following directly from (20) is the physical significance of the coefficient η_λ—a quantity equal to the reflectivity of an infinitely dense layer—as well as the Gurevich-Kubelka-Munk formula which finds greatest application in the spectroscopy of light-scattering media:

$$\frac{a_\lambda}{s_\lambda} = \frac{(1-R_{\lambda\infty})^2}{2R_{\lambda\infty}}. \quad (22)$$

We can obtain an equation similar to (22) for σ_λ from formula (9) by considering (22):

$$\frac{\sigma_\lambda}{s_\lambda} = \frac{1-R_{\lambda\infty}^2}{2R_{\lambda\infty}} \quad \text{or} \quad \frac{\sigma_\lambda}{a_\lambda} = \frac{1+R_{\lambda\infty}}{1-R_{\lambda\infty}}. \quad (23)$$

Having solved (14) and (16) simultaneously for $\sigma_\lambda l$ and keeping in mind (20), (22), and (23), we derive the following expressions which associate the optical properties of the medium with the thermal-radiation characteristics of the layer:

$$\sigma_\lambda l = \ln\left(\frac{1-R_\lambda R_{\lambda\infty}}{T_\lambda}\right), \quad (24)$$

$$s_\lambda l = \frac{2R_{\lambda\infty}}{1-R_{\lambda\infty}^2}\sigma_\lambda l, \quad (25)$$

as well as [3]

$$a_\lambda l = \frac{1-R_{\lambda\infty}}{1+R_{\lambda\infty}}\sigma_\lambda l. \quad (26)$$

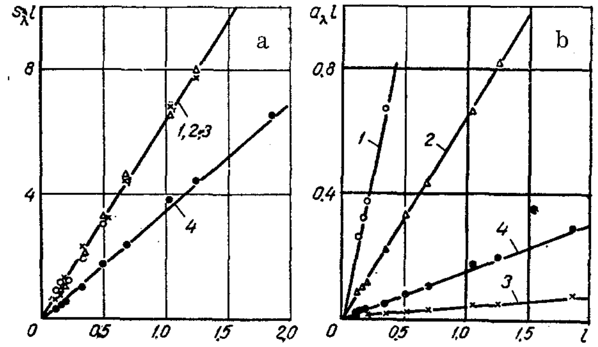

Fig. 2. Coefficients $s_\lambda l$ (a) and $a_\lambda l$ (b) of wood (pine) versus layer thickness l (mm) at various wavelengths: 1) λ = 0.5 μm; 2) 0.6; 3) 0.9; 4) 1.4.

Formulas (24), (25), and (26) permit us to determine the internal spectral optical characteristics of the material from the experimentally derived R_λ and T_λ of a layer of finite thickness l, as well as the $R_{\lambda\infty}$ of a layer that is infinitely dense from the optical standpoint. With consideration of (20), we can write formulas (14) and (16) in the following form, more convenient for calculation:

$$R_\lambda = R_{\lambda\infty}\frac{\exp(\sigma_\lambda l)-\exp(-\sigma_\lambda l)}{\exp(\sigma_\lambda l)-R_{\lambda\infty}^2\exp(-\sigma_\lambda l)}, \quad (27)$$

$$T_\lambda = \frac{1-R_{\lambda\infty}^2}{\exp(\sigma_\lambda l)-R_{\lambda\infty}^2\exp(-\sigma_\lambda l)}. \quad (28)$$

These formulas were tested for several powderlike materials which were densely packed (luminophores) [11], for colored glasses, etc. [3, 5, 8], and the possibility of their utilization was demonstrated. In [3] the applicability of these formulas was hypothesized for colloids and other turbid media.

To verify the derived relationships for the propagation of radiation within capillary-porous colloidal bodies, the authors developed a special adaptor for the SF-4 spectrophotometer by means of which it was possible to study the spectral thermal-radiation characteristics of various light-scattering materials. The adapter to the SF-4 spectrophotometer permits the simultaneous measurement of R_λ and T_λ in the spectral region from 0.4 to 1.5 μm for the case of normal incidence of monochromatic radiation onto the surface of the specimen.

The tests were carried out on such typical colloidal, capillary-porous colloidal, and capillary-porous materials as macaroni dough, fruit-candy pastille, raw potato, wood, and flour. As an example, in Fig. 2 we have presented $s_\lambda l$ and $a_\lambda l$ for wood (pine) in the air-dried state as a function of thickness for four wavelengths (0.5, 0.6, 0.9, and 1.4 μm), calculated from (24), (25), and (26) on the basis of experimental data. In the wavelength range 0.5–0.9 μm the scattering coefficient is independent of λ. Thus, the tests confirm the constancy of s_λ found for the luminophore powders [11] to be valid for the test material. However, this constancy of s_λ pertains only to that region of the spectrum in which the refractive index of the disperse me-

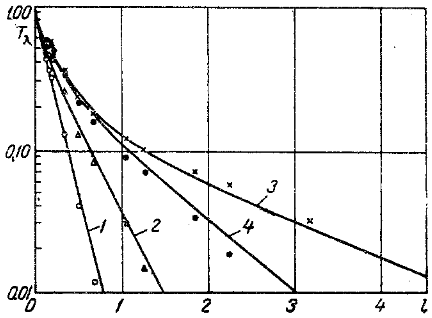

Fig. 3. Transmissivity T_λ of wood (pine) versus thickness l (mm) and various wavelengths: 1) $\lambda = 0.5\ \mu m$; 2) 0.6; 3) 0.9; 4) 1.4.

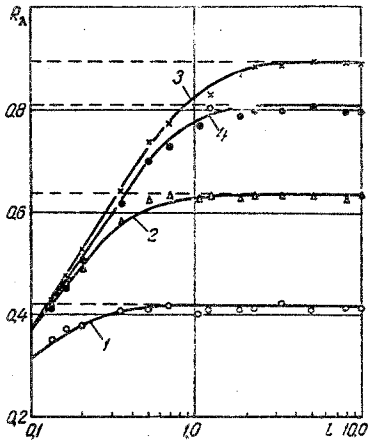

Fig. 4. Reflectivity R_λ of wood (pine) versus thickness l (mm) and various wavelengths: 1) $\lambda =$ $= 0.5\ \mu m$; 2) 0.6; 3) 0.9; 4) 1.4.

dium is approximately constant. However, with λ corresponding to 1.4 μm, we find, for example, that s_λ is smaller and a_λ is larger than when λ is equal to 0.9 μm. We know that the absorption band for water lies near 1.42 μm. Consequently, a_λ and s_λ are functions of the refractive and absorption indices for all components of the medium. Particularly important is the fact that for the indicated wavelengths, $a_\lambda l$ and $s_\lambda l$ as functions of the thickness of the layer are linear in nature. The values of T_λ and R_λ calculated from (27) and (28), and derived experimentally, are shown as functions of l in Figs. 3 and 4, from which it follows that the optical characteristics of capillary-porous colloidal bodies can be calculated on the basis of theoretical relationships. The latter are obtained by means of a method according to which the absorption and scattering of a layer of unit thickness are treated as the properties of the material itself. In this connection, the magnitude of the absorption coefficient for the unit layer—calculated from (26)—is in good agreement with the magnitude of the absorption coefficient for the material, this quantity having been determined experimentally.

On the basis of the above, we can draw the following conclusions.

1. We cannot use the Bouguer and Bouguer-Lambert laws to determine the distribution of radiation fluxes in absorbing and scattering media, because they do not provide for consideration of the co-current reflected flows.

2. Formulas have been derived for the distribution of monochromatic radiation fluxes and of the fraction of radiation energy absorbed by the unit layers through the thickness, and formulas have been derived to relate the optical properties with the thermal-radiation characteristics of capillary-porous colloidal bodies.

3. The cited relationships can be used for colloidal, capillary-porous colloidal, capillary-porous, powder-like, and similar light-scattering materials.

4. The limits of application for the derived relationships are considerably greater than those established earlier. The applicability of relationship (26) had earlier been restricted by the condition $a_\lambda l \leq 0.1$, while relationship (20) had been restricted by the condition $a_\lambda l \ll 1.0$ [3]. According to our experimental data and the results from the calculation of $a_\lambda l$ from (26), the validity of these relationships is retained within wide limits up to $a_\lambda l = 1.0$ (Fig. 2b).

NOTATION

R_λ, T_λ, and A_λ are the spectral reflectivity, transmissivity and absorptivity of the layer, thickness l; ε_λ is the spectral reflectivity of the real body; a_λ and s_λ are the spectral absorption and back scattering factors of a layer of unit thickness; $Q_{1,2}$ is the monochromatic radiation; q_+ and q_- are the discrete co-current spectral flows at a depth x; q_λ is the scalar value of the total density vector of monochromatic radiation at a depth x; $R_{\lambda\infty}$ is the spectral reflectivity of an infinitely dense optical layer.

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